

# $D_s(2317)$ and $D_s(2457)$ from HQET Sum Rules

Yuan-Ben Dai <sup>a</sup>, Chao-Shang Huang <sup>a</sup>, Chun Liu <sup>a</sup>, and Shi-Lin Zhu <sup>b</sup>

<sup>a</sup>*Institute of Theoretical Physics, Chinese Academy of Sciences, PO Box 2735, Beijing 100080, China*

<sup>b</sup>*Department of Physics, Peking University, Beijing 100871, China*

Within the framework of the heavy quark effective theory, QCD sum rules are used to calculate the masses of p-wave  $\bar{c}s$  states. The results for  $0^+$  and  $1^+$  states with the angular momentum of the light component  $j_l = 1/2$  are consistent with the experimental values for  $D_s(2317)$  and  $D_s(2457)$ , respectively.

PACS numbers: 12.39.Hg, 13.25.Hw, 13.25.Ft, 12.38.Lg

## I. INTRODUCTION

By the end of April, 2003, BaBar Collaboration announced the discovery of a  $D_s$  resonance state,  $D_s(2317)$  with a narrow width [1]. Its mass is surprisingly small compared to quark model expectations. Lately CLEO [2] confirmed it and announced a new state  $D_s(2457)$  which also appears in BaBar's data. BELLE confirmed both of the above results [3].

The quantum numbers and decay modes of these two states are given in the following table.

The experimental discovery of these two states has drawn a lot of theoretical attentions. The narrowness of these states is understood as follows. Their low masses forbid them to decay to  $D^{(*)}K$ . For  $D_s(2317)$  the next possible decay mode  $D_s\eta$  can not open either. But the decay mode  $D_s\pi^0$  is viable due to the small mixing between  $\eta$  and  $\pi^0$  through isospin-violation. The key point is to understand their low masses. Previously quark models estimated the  $0^+$  state to be over 100 MeV higher than  $D_s(2317)$  [4]. The model using the heavy quark mass expansion of the relativistic Bethe-Salpeter equation in [5] predicted a lower value 2.369 GeV of  $0^+$  mass which is still 50 MeV higher than the experimental data. They were interpreted as the parity conjugate states of the  $D_s(0^-, 1^-)$  mesons in the framework of chiral symmetry [6]. A four-quark state explanation to  $D_s(2317)$  was suggested in Ref. [7]. Ref. [8] argued that the low mass of  $D_s(2317)$  could arise from the mixing between  $DK$  continuum and the lowest scalar nonet. For other discussions, see *e.g.* Refs. [9].

In this talk we report our calculations of the masses of p-wave  $D_s$  states from heavy quark effective theory (HQET) sum rules [10]. S-wave, non-strange heavy mesons have been studied with HQET sum rules, *e.g.* in [11], and the strange ones in [12]. The p-wave, non-

strange heavy mesons have been studied up to the order of  $\mathcal{O}(1/m_Q)$  [13, 14, 15]. They were also analyzed in full QCD sum rules [16].

## II. HQET DESCRIPTION

Let us first describe the p-wave  $D_s$  states. There are four states:  $D_{s0}^*(0^+)$ ,  $D_{s1}^*(1^+)$ ,  $D_{s1}(1^+)$  and  $D_{s2}^*(2^+)$ . These kinds of heavy mesons can be systematically studied by HQET [17]. In the heavy quark limit,  $m_Q/\Lambda_{\text{QCD}} \rightarrow \infty$ , there is heavy quark spin-flavor symmetry. After the redefinition of the quark field,  $P_+Q(x) = \exp(-im_Qv \cdot x)h_v(x)$ , where  $P_+ = \frac{1}{2}(1 + \not{v})$ , the HQET Lagrangian for heavy quark is simplified to be

$$\mathcal{L}_{\text{HQET}} = \bar{h}_v i v \cdot D h_v. \quad (1)$$

in this limit. The heavy quark spin symmetry implies that the angular momentum of the light quark system  $j_l$  is a good quantum number. The four p-wave heavy mesons can be grouped into two doublets  $(D_{s0}^*, D_{s1}^*)$  and  $(D_{s1}, D_{s2}^*)$ .  $j_l = 1/2$  for  $(D_{s0}^*, D_{s1}^*)$  and  $j_l = 3/2$  for  $(D_{s1}, D_{s2}^*)$ . It is important to note that although  $D_{s1}^*$  and  $D_{s1}$  have same total quantum numbers, they are clearly separated in the heavy quark limit. Their mixing is at the order of  $1/m_c$ , which can be studied in HQET. In HQET, the meson masses are expanded as

$$M = m_Q + \bar{\Lambda}_s, \quad (2)$$

where  $\bar{\Lambda}_s$  are independent of heavy quark flavors in the heavy quark limit.

To study properties of hadrons with QCD sum rules, the interpolating currents carrying the same quantum numbers in the HQET as those of the hadrons should be used. For the p-wave heavy mesons, we write the currents as follows [13],

$$J = \frac{1}{\sqrt{2}}\bar{h}_v \Gamma D_\rho s, \quad (3)$$

TABLE I: Spin-parity numbers and decay modes

	$D_s(2317)$	$D_s(2457)$
$J^P$	$0^+$	$1^+$
Decays	$D_s\pi^0$	$D_s^*\pi^0$

with  $\Gamma$  denoting some  $\gamma$  matrices,  $\Gamma$  is found to be

$$\begin{aligned}\Gamma &= -\gamma_t^\rho & \text{for } D_{s0}^*, \\ \Gamma &= \gamma_5 \gamma_t^\mu \gamma_t^\rho & \text{for } D_{s1}^*, \\ \Gamma &= -\sqrt{\frac{3}{2}} \gamma_5 (g_t^{\mu\rho} - \frac{1}{3} \gamma_t^\mu \gamma_t^\rho) & \text{for } D_{s1}, \\ \Gamma &= \sqrt{\frac{1}{2}} (\gamma_t^\mu g_t^{\nu\rho} + \gamma_t^\nu g_t^{\mu\rho}) - \frac{1}{3} g_t^{\mu\nu} \gamma_t^\rho & \text{for } D_{s2}^*\end{aligned}\quad (4)$$

where  $\gamma_t^\mu \equiv \gamma^\mu - v^\mu$  (In the rest frame of the meson,  $\gamma_t^\mu = (0, \vec{\gamma})$ ) and  $g_t^{\mu\nu} \equiv g^{\mu\nu} - v^\mu v^\nu$ . The related decay constant  $f$  is defined as

$$\langle 0 | J^\dagger | D_s^{**}(v, \eta) \rangle \equiv f\eta, \quad (5)$$

where  $D_s^{**}$  stands for the p-wave meson with polarization tensor  $\eta$ , which is the lowest state coupling to  $J$ . We will give the sum rule calculations for masses of the mesons to the order of  $1/m_c$ .

### III. QCD SUM RULES

The QCD sum rule [18] is a nonperturbative method rooted in QCD itself. We start from the the Green function from which the hadron mass parameter can be calculated.

#### A. Green function

For calculating the masses, the Green function is written as

$$\Pi(\omega) = i \int d^4x e^{ik \cdot x} \langle 0 | T J^\dagger(x) J(0) | 0 \rangle, \quad (6)$$

with  $\omega = 2k \cdot v$ . It can be expressed in the hadronic language,

$$\Pi(\omega) = \frac{2f^2}{2\bar{\Lambda}_s - \omega} + \text{higherstates}. \quad (7)$$

The calculation of  $\Pi(\omega)$  involves perturbation part and condensation part. The condensates are included up to dimension five. The duality hypothesis is used to simulate the higher states by perturbative contribution above some threshold  $\omega_c$ . The sum rule is further improved by Borel transformation. The final sum rule is

$$2f^2 e^{-2\bar{\Lambda}_s/T} = \frac{1}{\pi} \int_{2m_s}^{\omega_c} d\nu \text{Im} \Pi^{\text{pert}}(\nu) e^{-\nu/T} + \hat{B}_T^\omega \Pi^{\text{cond}}(\omega), \quad (8)$$

#### B. Sum rules in the leading order

We found the sum rules in the leading order of the heavy quark expansion [10],

$$\begin{aligned}f^2 e^{-2\bar{\Lambda}_s/T} &= \frac{3}{64\pi^2} \int_{2m_s}^{\omega_c} (\nu^4 + 2m_s \nu^3 - 6m_s^2 \nu^2 \\ &\quad - 12m_s^3 \nu) e^{-\nu/T} d\nu - \frac{1}{16} m_0^2 \langle \bar{s}s \rangle \\ &\quad + \frac{3}{8} m_s^2 \langle \bar{s}s \rangle - \frac{m_s}{16\pi} \langle \alpha_s G^2 \rangle,\end{aligned}\quad (9)$$

for  $(0^+, 1^+)$  doublet. And

$$\begin{aligned}f^2 e^{-2\bar{\Lambda}_s/T} &= \frac{1}{64\pi^2} \int_{2m_s}^{\omega_c} (\nu^4 + 2m_s \nu^3 - 6m_s^2 \nu^2 \\ &\quad - 12m_s^3 \nu) e^{-\nu/T} d\nu - \frac{1}{12} m_0^2 \langle \bar{s}s \rangle \\ &\quad - \frac{1}{32} \langle \frac{\alpha_s}{\pi} G^2 \rangle T + \frac{1}{8} m_s^2 \langle \bar{s}s \rangle - \frac{m_s}{48\pi} \langle \frac{\alpha_s}{\pi} G^2 \rangle,\end{aligned}\quad (10)$$

for  $(1^+, 2^+)$  doublet. The strange quark mass  $m_s$  is kept in the above calculation. It should be noted that the strange quark condensate has a different value from up or down quarks.

#### C. Numerical analysis

In the numerical analysis, we require that the high-order power corrections be less than 30% of the perturbation term. This condition yields the minimum value  $T_{\min}$  of the allowed Borel parameter. We also require that the pole term, which is equal to the sum of the cut-off perturbative term and the condensation terms, is larger than 60% of the perturbative term, which leads to the maximum value  $T_{\max}$  of the allowed  $T$ .

We use  $m_s = 150$  MeV for the strange quark mass. The values of various QCD condensates are

$$\begin{aligned}\langle \bar{s}s \rangle &= -(0.8 \pm 0.1) * (0.24 \text{ GeV})^3, \\ \langle \alpha_s GG \rangle &= 0.038 \text{ GeV}^4, \\ m_0^2 &= 0.8 \text{ GeV}^2.\end{aligned}\quad (11)$$

We use  $\Lambda_{QCD} = 375$  MeV for three active flavors.

For  $(0^+, 1^+)$  doublet, the allowed interval of  $T$  is determined to be  $0.38 < T < 0.58$  GeV,

$$\bar{\Lambda}_s(j_l = \frac{1}{2}) = (0.86 \pm 0.10) \text{ GeV}, \quad (12)$$

where the central value corresponds to  $T = 0.52$  GeV and  $\omega_c = 2.9$  GeV. For  $(1^+, 2^+)$  doublet, the allowed interval of  $T$  is  $0.55 < T < 0.65$  GeV,

$$\bar{\Lambda}_s(j_l = \frac{3}{2}) = (0.83 \pm 0.10) \text{ GeV}, \quad (13)$$

where the central value corresponds to  $T = 0.62$  GeV and  $\omega_c = 3.0$  GeV. The errors of the above results refer to the variation within the stability windows and the uncertainty of  $\omega_c$ .

#### D. $1/m_c$ corrections

The  $1/m_c$  corrections to  $D_s^{**}$  masses come from the Lagrangian of that order, which is heavy quark symmetry breaking,

$$\mathcal{L}_{1/m} = \frac{1}{2m_Q} \mathcal{K} + \frac{1}{2m_Q} \mathcal{S}, \quad (14)$$

with the heavy quark kinetic operator  $\mathcal{K}$  and chromomagnetic operator  $\mathcal{S}$  being defined as

$$\mathcal{K} = \bar{h}_v (iD_t)^2 h_v, \mathcal{S} = \frac{g}{2} C_{mag}(m_Q/\mu) \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v, \quad (15)$$

where  $C_{mag} = \left(\frac{\alpha_s(m_Q)}{\alpha_s(\mu)}\right)^{3/\beta_0}$ ,  $\beta_0 = 11 - 2n_f/3$ . Note that  $\mathcal{S}$  violates both heavy quark flavor and spin symmetries.

The  $1/m_Q$  correction to the meson masses is written as

$$\delta M = -\frac{1}{4m_Q} (K + d_M C_{mag} \Sigma), \quad (16)$$

where  $K$  and  $d_M \Sigma$  are the mesonic matrix elements of  $\mathcal{K}$  and  $\mathcal{S}$ , respectively.  $d_M$  is  $(3, -1)$  for  $(0^+, 1^+)$  states, and is  $(5, -3)$  for  $(1^+, 2^+)$  states.

In this work, we adopt the method of Ref. [20] to use three-point correlation functions and double Borel transformation. The  $m_s$  terms are neglected at this order. Therefore, the sum rules are the same as those in [14]. The sum rule window is taken as the same as that for the leading order. The mixing of the two  $1^+$  states at this order, which was found to be small in [14], is neglected. The numerical results of  $K$  and  $\Sigma$  are obtained as follows [10],

$$\begin{aligned} K(j_l = \frac{1}{2}) &= (-1.60 \pm 0.30) \text{ GeV}^2, \\ \Sigma(j_l = \frac{1}{2}) &= (0.28 \pm 0.05) \text{ GeV}^2; \\ K(j_l = \frac{3}{2}) &= (-1.64 \pm 0.40) \text{ GeV}^2, \\ \Sigma(j_l = \frac{3}{2}) &= (0.058 \pm 0.01) \text{ GeV}^2. \end{aligned} \quad (17)$$

#### E. Final results

We present our final results in terms of the weighted average mass and the mass splitting of each doublet,

$$\begin{aligned} \frac{1}{4} (m_{D_{s0}^*} + 3m_{D_{s1}^*}) &= m_c + (0.86 \pm 0.10) \\ &\quad + \frac{1}{m_c} [(0.40 \pm 0.08) \text{ GeV}^2], \\ m_{D_{s1}^*} - m_{D_{s0}^*} &= \frac{1}{m_c} [(0.28 \pm 0.05) \text{ GeV}^2]. \end{aligned} \quad (18)$$

And

$$\begin{aligned} \frac{1}{8} (3m_{D_{s1}} + 5m_{D_{s2}^*}) &= m_c + (0.83 \pm 0.10) \\ &\quad + \frac{1}{m_c} [(0.41 \pm 0.10) \text{ GeV}^2], \\ m_{D_{s2}^*} - m_{D_{s1}} &= \frac{1}{m_c} [(0.116 \pm 0.06) \text{ GeV}^2]. \end{aligned} \quad (19)$$

We obtain the value of  $m_c = 1.44$  GeV from the quantity in the first equation in (19), which is well measured. This value is in good agreement with that fitting ground state charm hadrons [11, 19]. Then we predict,

$$\begin{aligned} m_{D_{s2}^*} - m_{D_{s1}} &= (0.080 \pm 0.042) \text{ GeV} & \text{Exp 37 MeV}, \\ \frac{1}{4} (m_{D_{s0}^*} + 3m_{D_{s1}^*}) &= (2.57 \pm 0.12) \text{ GeV} & \text{Exp 2.42 GeV}, \\ m_{D_{s1}^*} - m_{D_{s0}^*} &= (0.19 \pm 0.04) \text{ GeV} & \text{Exp 143 MeV}. \end{aligned} \quad (20)$$

The experimental values have been also given for comparison. The consistency can be seen. And

$$m_{D_{s0}^*} = 2.42 \pm 0.13 \text{ GeV}, \quad \text{Exp: 2.317 GeV}. \quad (21)$$

The experimental values have been also given for comparison. It can be seen that the results of calculations are consistent with the experimental data within the uncertainties.

## IV. SUMMARY AND DISCUSSION

The HQET sum rules have been used to the order of  $1/m_c$  for calculating p-wave  $D_s$  meson masses. Within uncertainties of QCD sum rules, our results for  $(0^+, 1^+)$  states are consistent with experimental data for  $D_s(2317)$  and  $D_s(2457)$ . But the result for the central value of the mass  $D_{s0}^*$  meson is still 100 MeV higher than  $D_s(2317)$ . The stability of the sum rules obtained is not as good as that for the ground states. So the predictions for the masses have large uncertainties, which are estimated as the errors given. We would like to emphasize that the result for the mass splitting between these two states agrees with the experimental value within relatively small uncertainty and this result is not sensitive to the window taken for the sum rule for  $\Sigma(j_l = \frac{1}{2})$ .

Finally in the same way, we have also calculated the non-strange p-wave  $D$  meson masses [10]. The results are consistent with experimental data within theoretical uncertainties.

The repulsion between the  $D_s(0^+)$ ,  $D_s(1^+)$  states and the  $DK$  and  $DK^*$  continuum may help to lower their masses. In the framework of the sum rule this effect should come from the contribution from the  $DK$  and  $DK^*$  continuum to the dispersion integral.

## Acknowledgments

The speaker is grateful to Profs. C.W. Kim and Eung Jin Chun for kind hospitality and support during his stay

in KIAS, Seoul. This project was supported by the National Natural Science Foundation of China, Ministry of Education of China and BEPC Opening Project.

---

[1] Babar Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **90**, 242001 (2003).

[2] CLEO Collaboration, D. Besson *et al.*, hep-ex/0305017.

[3] BELLE Collaboration, P. Krokovny *et al.*, hep-ex/0308019.

[4] S. Godfrey and R. Kokoski, Phys. Rev. **D 43**, 1679 (1991);  
S. Godfrey and N. Isgur, Phys. Rev. **D 32**, 189 (1985);  
M. Di Pierro and E. Eichten, Phys. Rev. **D 64**, 114004 (2001).

[5] Y.-B. Dai, C.-S. Huang and H.-Y. Jin, Phys. Lett. **B 331**, 174 (1994).

[6] W. A. Bardeen, E. J. Eichten, and C. T. Hill, Phys. Rev. **D 68**, 054024 (2003); D. Ebert, T. Feldmann, and H. Reinhardt; Phys. Lett. **B 388**, 154 (1996); C.-S. Huang, Phys. Rev. **D 51**, 1243 (1995).

[7] H.-Y. Cheng and W.-S. Hou, Phys. Lett. **B 566**, 193 (2003); T. Barnes, F. E. Close, H. J. Lipkin, Phys. Rev. **D 68**, 054006 (2003).

[8] E. van Beveren and G. Rupp, Phys. Rev. Lett. **91**, 012003 (2003).

[9] R. N. Cahn and J. D. Jackson, Phys. Rev. **D 68**, 037502 (2003); G. S. Bali, *ibid.* **68**, 071501 (2003); K. Terasaki, *ibid.* **68**, 011501 (2003); A. Deandrea, G. Nardulli, A.D. Polosa, *ibid.* **68**, 097501 (2003); A. P. Szczepaniak, Phys. Lett. **B 567**, 23 (2003); S. Godfrey, *ibid.* **568**, 254 (2003); P. Colangelo and F. De Fazio, *ibid.* **570**, 180 (2003); UKQCD Collaboration, A. Dougall *et al.*, *ibid.* **569**, 41 (2003); T. E. Browder, S. Pakvasa, A. A. Petrov, hep-ph/0307054; C.-H. Chen, H.-n. Li, hep-ph/0307075; M. Sadzikowski, hep-ph/0307084; M. A. Nowak, M. Rho, and I. Zahed, hep-ph/0307102; E. E. Kolomeitsev, M.F.M. Lutz, hep-ph/0307133; W. Lucha, F. F. Schoberl, hep-ph/0309341.

[10] Y.-B. Dai, C.-S. Huang, C. Liu and S.-L. Zhu, to appear in Phys. Rev. **D**, hep-ph/0306274.

[11] For a review, see M. Neubert, Phys. Rep. **245**, 259 (1994).

[12] T. Huang and C.-W. Luo, Phys. Rev. **D 50**, 5775 (1994).

[13] Y.-B. Dai, C.-S. Huang, M.-Q. Huang and C. Liu, Phys. Lett. **B 390**, 350 (1997);  
Y.-B. Dai, C.-S. Huang and M.-Q. Huang, Phys. Rev. **D 55**, 5719 (1997).

[14] Y.-B. Dai and S.-L. Zhu, Phys. Rev. **D 58**, 074009 (1998).

[15] P. Colangelo, F. De Fazio and N. Paver, Phys. Rev. **D 58**, 116005 (1998).

[16] P. Colangelo, G. Nardulli, A.A. Ovchinnikov and N. Paver, Phys. Lett. **B 269**, 201 (1991); P. Colangelo, G. Nardulli, and N. Paver, *ibid.* **293**, 207 (1992).

[17] For a review, see N. Isgur and M.B. Wise, in *B Decays* ed. S. Stone, 2nd ed. (World Scientific, Singapore, 1994).

[18] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B 147**, 358, 448 (1979).

[19] Y.-B. Dai, C.-S. Huang, C. Liu and C.-D. Lü, Phys. Lett. **B 371**, 99 (1996);  
Y.-B. Dai, C.-S. Huang, M.-Q. Huang and C. Liu, Phys. Lett. **B 387**, 379 (1996).

[20] P. Ball and V. M. Braun, Phys. Rev. **D 49**, 2472 (1994).